# On the parameterizations of the special unitary group SU(4) and related double cosets

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A nice structure of Lie groups due to their dual nature of being a group and a manifold simultaneously reveals in a rich variety of different types of coordinates on the group manifold. Among them traditionally used parameterizations in mathematics and theoretical physics are those related to the case when coordinates on the group manifold provide the projected coordinates on the corresponding quotients or bi-quotients obtained as a result of group action. There is a big amount of general results on the topological properties [1] and explicit constructions of different parameterization of compact Lie groups well adapted to either cosets or the double cosets description. Among them one can point out the classical results based on the Cartan involution [2], their generalization associated to a pair of the non-commutative involutions [3], and classification of all possible Kobayashi triples of the form $(U(n), U(p) \times U(q), U(n_1) \times U(n_2) \times \cdots \times U(n_l))$, with $n = p + q = n_1 + n_2 + \cdots + n_l$ [5].

In the present report we are not aiming to discuss the general theory of double coset description, but our less ambitious goal is to tackle the issue of parameterizations for the special unitary group SU(4) adapted to the description of double coset related to the triplet $(SU(4), SU(2) \times SU(2), T^3)$ comprised of the maximal torus $T^3$ and the subgroup $SU(2) \times SU(2) \subset SU(4)$. Based on the suggested "coordinatization", the Riemannian bi-invariant metric on group manifold is derived and the corresponding volume 15-form is calculated.

## References

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